Corporate Finance 2 - Formeln

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1 Basics

1.1 Fixed-Investment Model

$$\begin{aligned} \text{I} &= \text{Fixed Investment} \\ \text{A} &= \text{Borrower assets} \\ \text{I - A} &= \text{Money needed from Lender to implement project} \\ \text{B} &= \text{Private benefit (for Borrower)} \end{aligned}$$

$$\text{Project Income} = \begin{cases} R > 0, & \text{if project succeeds} \\ 0, & \text{if project fails} \end{cases}$$

The project income is distributed to lender and borrower $R_l + R_b = R$: Lender gets R_l, while borrower gets R_b. Entrepreneur (Borrower) has initially assets (net worth) $A < I$.

Borrower behaving yields project success probability p_H while misbehaving yields a lower probability p_L and private benefit $B > 0$.

$$\Delta p = p_H - p_L$$

Zero profit constraint for lender, assumption is borrower behaves:

$$p_H R_l = I - A$$

In case of success lender gets

$$R_l = \frac{I - A}{p_H}$$

and borrower gets $R_b = R - R_l$ with a borrowers utility in case of success of

$$U_b = p_H R_b - A \text{ or } p_H R - I.$$

Rate of interest r:

$$R_l = (1 + r)(I - A) \text{ or } 1 + r = \frac{1}{p_H}$$

Project is viable only in the absence of moral hazard:
Positive NPV if entrepreneur behaves

$$p_H R - I > 0,$$

but negative NPV, if she does not,

$$p_L R - I + B < 0.$$

Borrower behaves if incentive compatibility constraint (IC_b) is satisfied:

$$p_H R_b \geq p_L R_b + B$$
or
$$(\Delta p) R_b \geq B \qquad IC_b$$

Agency cost (AC) is

$$AC = \frac{B}{\Delta p}.$$

BV: Expected pledgeable income is

$$P = p_H(R - \frac{B}{\Delta p}) = p_H(R - AC).$$

LV: Expected pledgeable income must exceed the lenders investment (IR_l):

$$P = p_H(R - \frac{B}{\Delta p}) \geq I - A \qquad IR_l$$

Necessary condition for financing is therefore (take IR_l and solve for A)

$$A \geq \bar{A} = p_H \frac{B}{\Delta p} - p_H R + I.$$

Equity multiplier is $\frac{I}{A}$. Value of debt is $I - A$ and value of equity is $U_b + A$. Shadow value of equity is v with value of equity $= v * A$.

1.2 Sequence of events

$t = 0$ Loan Agreement

* Investment ($I - A$)
* Moral Hazard (p_H or p_L; B)

$t = 1$ Outcome (R_l; R_b)

1.3 Continuous-Investment Model

$$(\Delta p)R_b \geq BI \qquad IC_b$$

$$p_H(RI - R_b) \geq I - A \qquad IR_l$$

Outside financing capacity I: Substitute IC_b into IR_l and solve for I.

$$I = \frac{A}{1 - p_H(R - \frac{B}{\Delta p})} \tag{1}$$

Equity multiplier k is $I = kA$.

2 Collaterals

Assumption: Borrower has no cash ($A = 0$). Investment is used to purchase an Asset with value A to entrepreneur (borrower) and value A' to lender with $A' \leq A$. The deadweight loss, if the asset is seized, is $A - A'$.

$$y_S = \text{Prob. borrower keeps asset in case of success}$$
$$y_F = \text{Prob. borrower keeps asset in case of failure}$$

Borrowers utility in case of success:

$$U_b = p_H(R_b + y_S A) + (1 - p_H)y_F A$$

$$\Delta p[R_b + (y_S - y_F)A] \geq B \qquad IC_b$$

$$p_H[R_l + (1 - y_S)A'] + (1 - p_H)(1 - y_F)A' \geq I \qquad IR_l$$

Expected pledgeable income:

$$p_H\left(R - \frac{B}{\Delta p}\right) + A'$$

Ranking borrowers:

- **Strong** No collateral: $y_S = y_F = 1, R_b > 0$
- **Intermediate** Collateral in case of failure: $y_S = 1, y_F \leq 1, R_b \geq 0$
- **Weak** Collateral in case of failure and success: $y_S \leq 1, y_F = 0, R_b = 0$

3 Summary: Monitoring vs. Advising

- **Monitoring**: reduces private benefit from B to $b < B$, has certain cost c
- **Advising**: increases probability of success (from p_H to $p_H + q_H$), increases the project utility, decreases pledgeable income

4 Monitoring

$$R_m = \text{Reward to monitor}$$

Incentive for monitoring:

$$(\Delta p) R_m \geq c$$

Monitor's investment contribution:

$$I_m = \frac{p_L}{\Delta p} c$$

Project is funded when

$$p_H \left(R - \frac{b}{\Delta p} \right) \geq I - A + c.$$

5 Advising

Probability of success is $p + q$ with $p \in \{p_H, p_L\}$ and $q \in \{q_H, q_L = 0\}$.

Advising increases surplus: $(\Delta q) R = q_H R \geq c$

Incentive for advising:

$$(\Delta q) R_m \geq c$$

Advisor's investment contribution:

$$I_m = (p_H + q_H) \frac{c}{\Delta q} - c$$

Pledgeable income:

$$(p_H + q_H) \left(R - \frac{B}{\Delta p} - \frac{c}{\Delta q} \right) \overset{!}{\geq} I - A - I_m$$

Entrepreneurs utility:

$$U_b = (p_H + q_H)R_b - A = (p_H + q_H)R - I - c$$

Project is funded when

$$(p_H + q_H)\left(R - \frac{B}{\Delta p}\right) \geq I - A + c.$$

Entrepreneur always obtains advisory service, if he can afford it.

6 Corporate Finance under Asymmetric Information

Borrower has no funds ($A = 0$). There are two borrower types: Good borrower with probability of success p and bad borrower with probability q. It holds $p > q$ and $pR > I$ (at least the good type is creditworthy).

There are two subcases: $pR > I > qR$ (only good type is creditworthy) and $pR > qR > I$ (both types are creditworthy). Additionally α probability of borrower being good type and $1 - \alpha$ of borrower being bad type.

$$m = \alpha p + (1 - \alpha)q$$

6.1 Symmetric Information

(in case of success)

Good borrower gets R_b^G with $p(R - R_b^G) = I$.

If $qR < I$ bad borrower does not invest, because NPV $qR - I$ is negative.
If $qR > I$ bad borrower gets R_b^B with $q(R - R_b^B) = I$.

6.2 Asymmetric Information

Investors profit is

$$m(R - R_b) - I$$

Measure of adverse selection is

$$\chi = (1 - \alpha) \left(\frac{p - q}{p} \right)$$

with $mR \geq I \mathrel{\widehat{=}} [1 - \chi]pR \geq I$.

No lending ($mR < I$)

Bad borrower is always not creditworthy. Market breaks down and there is ***under-investment***.

Happens when probability of getting a bad type is high enough ($\alpha < \alpha^*$). Calculate $mR \stackrel{!}{=} 0$ and solve for α^*.

Lending ($mR \geq I$)

Both types are creditworthy or bad type is not creditworthy (in this case there is ***over-investment***.

Borrowers compensation R_b is

$$m(R - R_b) = I$$

Investors make money on good type and lose money on bad type: ***Cross-subsidization***.

Cost for good borrower: $\frac{\chi}{1-\chi} I$.

7 Investment with reinvestment need

Sequence of events

$t = 0$ Loan Agreement and Investment $(I - A)$

$t = 1$ Short term income $r > 0$. Reinvestment need ρ.

$\downarrow$ (If reinvestment)

* Moral Hazard ($p = p_H$ or p_L)

$t = 2$ Success (profit R) with probability p
Failure (profit 0) with probability $1 - p$.

Reinvestment amount ρ is ex-ante unknown and has cumulative distribution function $F(\rho)$ with density $f(\rho)$.

A cutoff reinvestment amount ρ^* exists and it is optimal to continue when $\rho \leq \rho^*$. Probability of continuation is $Pr(\rho \leq \rho^*) = F(\rho^*)$.

$$(\Delta p) R_b \geq B \qquad IC_b$$

Investors breakeven condition is

$$[(r - r_b) + F(\rho^*) p_H (R - R_b)] = I - A + \int_0^{\rho^*} \rho f(\rho) d\rho \qquad IR_l$$

with $R_b \geq \frac{B}{\Delta p}$ and $r_b \geq 0$.

$$\begin{aligned} U_b &= [r + F(\rho^*) p_H R] - I - \int_0^{\rho^*} \rho f(\rho) d\rho \\ &= \text{Expected Revenue - Total Investment} \end{aligned}$$

Pledgeable income $P(\rho^*)$:

$$P(\rho^*) = [r + F(\rho^*) p_H \left(R - \frac{B}{\Delta p} \right)] - \int_0^{\rho^*} \rho f(\rho) d\rho$$

Expected income increases until $\rho^* = p_H R$ and pledgeable income increases until $\rho^* = p_H (R - \frac{B}{\Delta p})$.

Set of contracts
Short term debt r, credit line up to ρ^* and (long term) debt $D = p_H (R - \frac{B}{\Delta p})$.

Case 1: Efficient liquidation ($P(p_H R) \geq I - A$)

Optimal: $\rho^* = p_H R$, calculate r_b and R_b using IR_l.

Case 2: Inefficient liquidation ($P(p_H R) < I - A \leq P(p_H[R - \frac{B}{\Delta p}])$)

Optimal: $r_b = 0$ and $R_b = \frac{B}{\Delta p}$, calculate ρ^* using IR_l.

Case 3: No financing ($P(p_H[R - \frac{B}{\Delta p}]) < I - A$)

Funding is not feasible.

8 Stock Price Reaction to Equity Issuance

Entrepreneur owns project with probability p or q and reward R in case of success (without further investment). There is probability α of the project being p and $1 - \alpha$ of q. Assets in place are undervalued if probability of success is p and overvalued if probability is q.

There exists a posibility to increase the probabilities of success with cost I and $\tau R > I$. Probabilities become $p + \tau$ and $q + \tau$. Entrepreneur then offers stake R_1 with $(m + \tau)R_1 = I$.

Good borrower only issues when $(p + \tau)(R - R_1) \geq pR$.

$$\tau R - I \geq \frac{\chi_\tau}{1 - \chi_\tau} I \qquad (*)$$

8.1 Pooling equilibrium

If $(*)$ holds both types issue equity. There is no stock price reaction. Before and after total stock value is $(m + \tau)R - I$.

8.2 Separating equilibrium

If $(*)$ is violated only the bad type issues equity. Lenders then demand a higher stake $R_1^B > R_1$ with $(q+\tau)R_1^B = I$. The good borrower does not want to raise funds because $(p+\tau)(R - R_1^B) < pR$.

Before total stock value is $V_0 = \alpha pR + (1-\alpha)[(q+\tau)R - I]$.
After total stock value is $V_1 = (q+\tau)R - I$.

9 Product Markets

$$\text{Profit of a firm} = \begin{cases} M & \text{if the only firm to succeed,} \\ D & \text{if both firms succeed,} \\ 0 & \text{if it fails,} \end{cases}$$

with $M \geq D \geq 0$ and $p_H M > I$. Predatory action is available, which has a cost k and reduces firm 2 profit from A to a with $a < A$.

Firm chooses predatory action if $k < p_H^2(M - D)$.

Return R is $p_H D + (1 - p_H)M$.

Pledgeable income is: $p_H(R - \frac{B}{\Delta p})$.

Financing agreement: z_F probability of obtaining date-1 financing when date 0 project failed. z_S date-1 probability when date 0 project succeeded.

$$\text{Rewards for borrower at date 2} = \begin{cases} R_b^S & \text{date 1 project succeeded} \\ R_b^F & \text{date 1 project failed} \end{cases}$$

$$\text{Expected continuation payoffs} = \begin{cases} R_b^{S*} = z^S R_b^S & \text{date 1 project succeeded} \\ R_b^{F*} = z^F R_b^F & \text{date 1 project failed} \end{cases}$$

Incentive Constraint $IC_b : (\Delta p)(R_b^{S*} - R_b^{F*}) \geq B_0$.

Investor break-event constraint $IR_L : U_b(z^S) - R_b^{S*} + A \geq 0$.

Avoiding predation

When $D \geq (z^S - z^F)(M - D)$ predation is avoided.

NPV is $U_b(z^S) = D - I + z^S(D - I)$.

Low-A if $U_b(1) - \frac{B}{\Delta p} + A < 0$.

10 Payout policy

Fix payment r at date 1. Can be reinvested or used as dividend.

Investment is only useful if bad type.

Good type does not invest: $r + \tau_G(0)R \geq \tau_G(r)R$.
Bad type does invest: $r + \tau_B(0)R \leq \tau_B(r)R$.

NPV is then: $\alpha[r + (p_H + \tau_G(0))R] + (1-\alpha)[(p_H + \tau_B(r))R]$.

Pledgeable income: $\alpha[r + (p_H + \tau_G(0))R - (p_H + \tau_G(r))\frac{B}{\Delta p}] +$
$(1-\alpha)[(p_H + \tau_B(r))(R - \frac{B}{\Delta p}]$.

$$\text{Dividend } IC_b = \begin{cases} r_b^r + [p_H + \tau_G(0)]R_b^r \geq [p_H + \tau_G(r)]R_b^0 & \text{good type} \\ r_b^r + [p_H + \tau_B(0)]R_b^r \leq [p_H + \tau_B(r)]R_b^0 & \text{bad type} \end{cases}$$

11 Misc

- **Hidden action (Moral Hazard)**
  Example: An entrepreneur may make decisions that provide a private benefit.
- **Hidden characteristics (Adverse selection)**
  Example: An entrepreneur may be better informed about a project's profitability than investors. Asymmetric information

11.1 Pecking order hypothesis

Firms prefer to use "internal finance" (initial equity, retained earnings) to finance their investments. If internal finance is an insufficient source of funds and external finance is required, firms first issue debt, the n hybrid securities such as convertibles, and finally equity.